

Nonlinear Kinetic Dynamics of Magnetized Weibel Instability

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Abstract. Kinetic numerical simulations of the evolution of the Weibel instability during the full nonlinear regime are presented. The formation of strong distortions in the electron distribution function resulting in formation of strong peaks in it and their influence on the resulting electrostatic waves are shown.

1. Introduction

The process of magnetic field generation in a plasma is one of the most important problems for both laboratory and astrophysical plasmas. Several mechanisms of magnetic field generation have been analyzed that are effective in different plasma regimes. At high frequencies, i.e., on short electron time scales over which the ions can be assumed to be immobile and collisions are unimportant, the Weibel instability can efficiently generate magnetic fields in plasmas with an anisotropic electron temperature distribution with zero real frequencies.

The Weibel instability is an electromagnetic plasma instability driven by the presence of temperature or electron momentum anisotropy [1].

In this paper we discuss the long time, kinetic, nonlinear evolution of the Weibel instability in an electron-ion plasma by using Vlasov-Maxwell simulations in the 1D-2V phase space. Relativistic effects on the plasma dynamics will be not considered, which become significant only for very high beam speeds.

The Weibel instability developed due to the counter streaming of electrons have been studied and showed that other than the electromagnetic instability, there is also an electrostatic limit that corresponds to the well known two-stream instabilities. For more results we refer to Refs. [2, 3]. In this paper instead, we assumed temperature anisotropy to study Weibel instability and will show, even in this case too, some kind of electrostatic instability develops due to acceleration of the particles.

The Weibel instability emerges from a wide white noise spectrum of wavelengths around the electron skin depth, d_e . This length plays the role of the natural scale length for the instability and will be used in the following as the normalizing length.

In general, since the growth rate of the Weibel instability is comparable to the electron plasma frequency, and also the electromagnetic nature of the instability, this study of the Weibel instability in the nonrelativistic limit has been limited so far to the electron dynamics. This approximation, which considers the ions as fixed neutralizing background, is valid during the linear and very early nonlinear phase of the instability. On the other hand, for longer times, the ion response cannot be neglected.

The paper is organized as follows. In the next section we will introduce the governing equations and the initial conditions. The numerical simulations are presented in section III. Conclusions follow in section IV.

2. The Equations

The kinetic evolution of an electron-ion collisionless plasma is described by the Vlasov equation in the phase space, coupled self-consistently to the Maxwell equations. We have used 1D-2V (x, v_x, v_y) phase space, with periodic boundary condition along x direction, unidirectional magnetic field B_z and a two-component electric field $\mathbf{E} = (E_x, E_y)$.

The Vlasov-Maxwell equations are normalized by the following characteristic quantities: the electron mass m_e , the characteristic electron density \bar{n}_e , the speed of light c , the electron plasma frequency $\omega_{pe} = (4\pi\bar{n}_e e^2/m_e)$, the electron skin depth $d_e = c/\omega_{pe}$ and the characteristic fields $\bar{E} = \bar{B} = m_e \omega_{pe} c/e$. In dimensionless limits the Vlasov-Maxwell equations for electrons read as

$$\frac{\partial f_e}{\partial t} + \mathbf{v} \cdot \frac{\partial f_e}{\partial \mathbf{x}} + (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_e}{\partial \mathbf{v}} = 0 \quad (1)$$

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x}, \quad \frac{\partial E_y}{\partial t} = -\frac{\partial B_z}{\partial x} + J_y \quad (2)$$

where $f_e(x, v_x, v_y, t)$ is the 1D-2V electron distribution function (edf). The Poisson equation given by $\frac{\partial^2 \phi}{\partial x^2} = (n_i - n_e)$, where, ϕ is the potential and n_i and n_e are ion and electron density respectively.

We integrate Vlasov equation in the interval $L_x = 2\pi/k$ to have the total length of the box large enough to include the most unstable mode in the simulation. The initial dimensionless distribution function considered is:

$$f_e(x, v_x, v_y, 0) = \left(\frac{n_e}{\pi v_{th}^2} \sqrt{\frac{T_y}{T_x}} \right) \exp \left[\frac{1}{v_{th}^2} \left(-v_x^2 - \frac{T_x}{T_y} v_y^2 \right) \right] \left[1 + \epsilon \sum_{n=1}^{300} \cos(kx + \phi(k)) \right] \quad (3)$$

where, L_x is the length in the x direction, v_{th} is the thermal velocity of the electrons and T_x and T_y are the temperatures along x and y direction respectively, the modes of the wave, $k = 2\pi n/L_x$, and ϵ is the perturbation amplitude. To include all the modes possible we choose 300 as the summation limit. At $t = 0$ we introduce also a perturbation on the magnetic field:

$$B_z = a_{in} \sum_{n=1}^{300} [\cos(k_n x + \phi(k))] \quad (4)$$

where, a_{in} is the magnetic field amplitude and is independent of the mode.

3. Numerical Simulations

We have performed the numerical simulation (see Ref. [4] for the numerical algorithm) of the Weibel instability in plasma, by integrating numerically Eqs.(1)-(2) with the initial conditions given by Eqs.(3)-(4). In particular, the electron and proton distribution function is advanced in time using the electromagnetic splitting method in the 1D-2V limit. The Maxwell equation are integrated in time by using the third order Adams - Bashforth algorithm and standard Fast

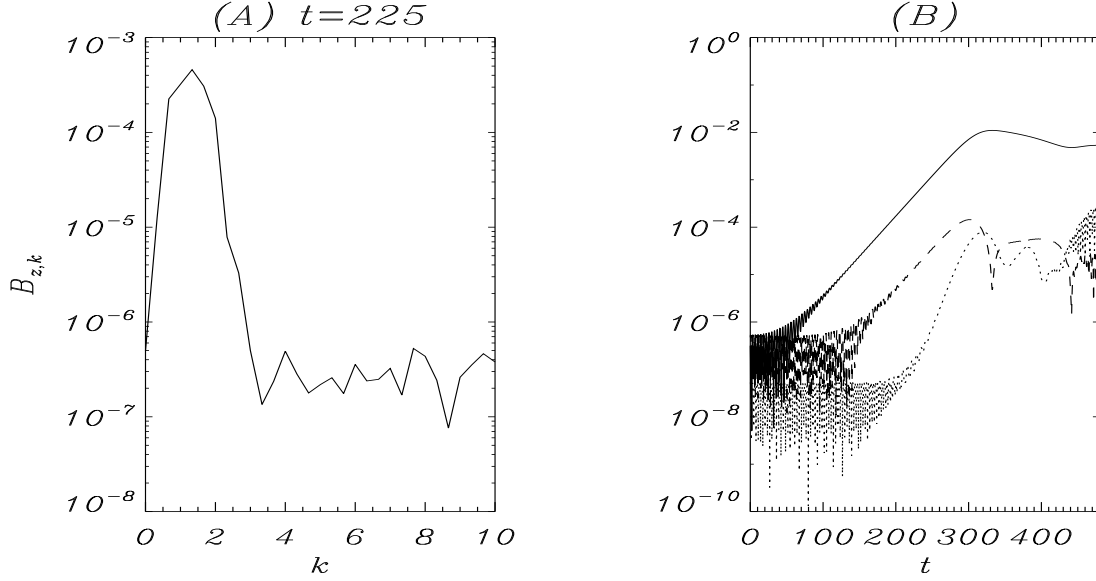


Figure 1. Fig(A) shows the Fourier transform of the magnetic field $B_{z,k}$ vs k at $t = 225$ showing the most unstable mode at $k = 1.25$. Fig(B) plots the time evolution of $B_{z,k}$ (solid lines), $E_{x,k}$ (dotted lines), $E_{y,k}$ (dashed lines) for $k = 1.25$.

Fourier Transform technique are applied to calculate the space derivatives. The temperature anisotropy considered is $T_y/T_x = 12$. We have performed the simulation with higher and lower temperature anisotropies also. But with lower temperature anisotropies the growth of the magnetic field is not strong enough to see the features we are interested in and also take longer time to run the simulation to reach the nonlinear regime. Nothing interesting is found at higher temperature anisotropies too. Hence, for the sake of qualitative agreement we assumed the particular case of anisotropy with $T_y/T_x = 12$. The thermal velocity is taken as $v_{th} = 0.02$, corresponding a plasma with a non-relativistic temperature. The perturbation amplitude for each wave is $a_{in} = 10^{-4}$. Initially, we consider a field free plasma, hence $\langle B_z \rangle = 0$. The number of points in the x and v directions are $N_x = 600$ and $N_{v_x} = 90$, $N_{v_y} = 90$. The time step is $\Delta t = 0.005$. The evolution of the system is investigated up to $t = 480\omega_{pe}^{-1}$. The maximal velocity which defines the velocity interval is set to $v_{x_{max}} = 0.1$ and $v_{y_{max}} = 0.3$ in the x and y directions respectively such that $-0.1 \leq v_x \leq 0.1$ and $-0.3 \leq v_y \leq 0.3$. The length of the space domain is $L_x = 6\pi$. The space discretization in x is $dx = 0.03d_e$, where d_e is the electron skin depth.

In Fig.1 (A) we plot the Fourier amplitudes of the magnetic field $B_{z,k}$ vs k at $t = 225$. We see the modes in the interval $0.1 \leq k \leq 3$ have grown with a maximum growth rate corresponding to $k = 1.25$. Note that a small portion of interval in k is taken. In (B) we plot the Fourier amplitude of the magnetic field and of the x and y components of the electric field. In the linear phase, $t > 100$, the magnetic field grows exponentially, corresponding to a growth rate $\gamma = 0.0390$. During this phase the Fourier amplitudes of the electrostatic field $E_{x,k}$ remains negligible. However from $t = 190$ to $t < 300$, the electrostatic field starts to grow at a very rapid rate, of the order of twice the linear growth rate of the instability, $\gamma_{es} = 2\gamma$ (γ_{es} is the electrostatic growth rate), a clear signature of the beginning of the non linear regime. At later time, $t > 300$ the magnetic field saturates. The electrostatic field levels off at $t > 300$ due to particle trapping in phase space.

In Fig.2 we plot the electron distribution function(edf) in (v_x, v_y) space at time $t =$

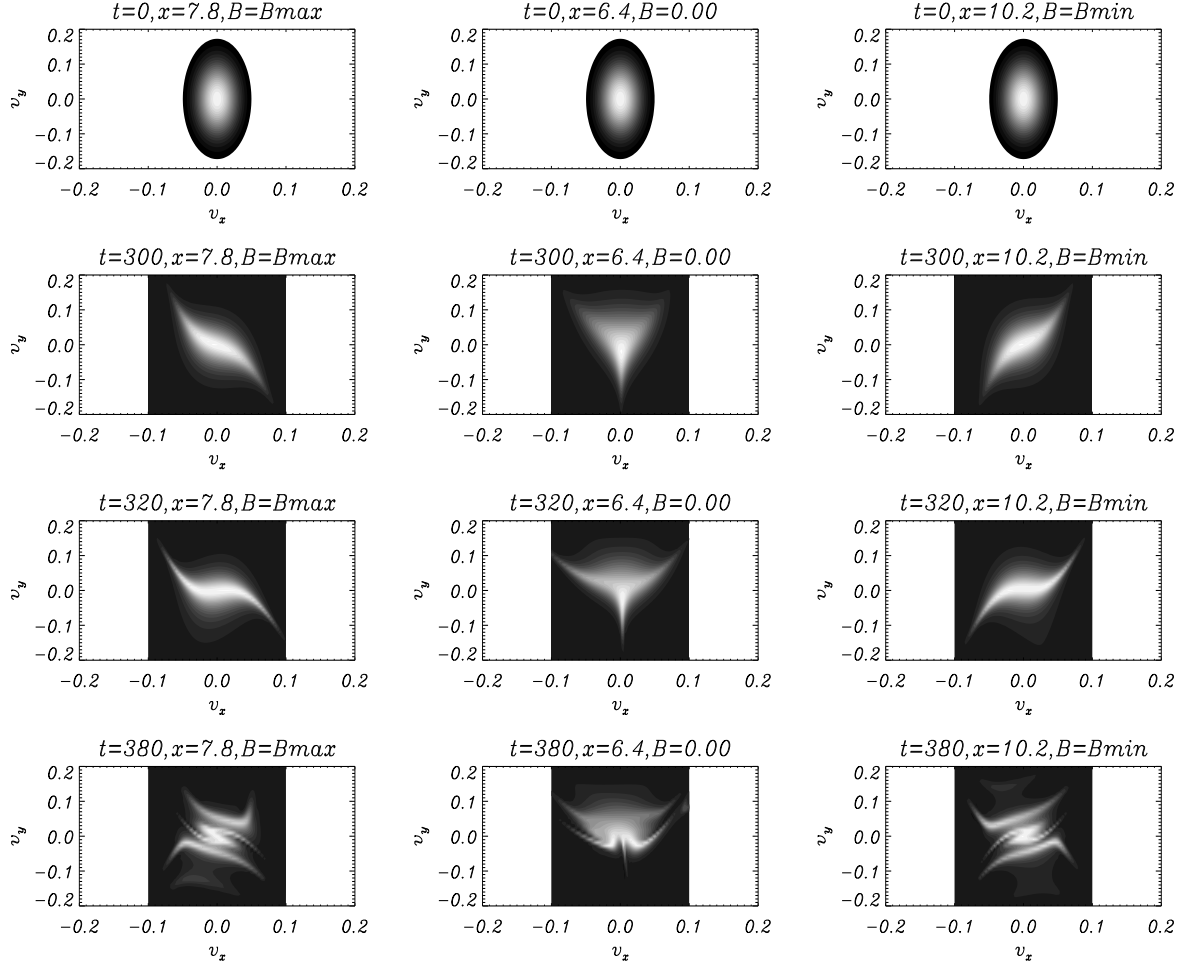


Figure 2. Contour plot of the distribution function at $t = 0, 300, 320, 380$ at $x = 7.8, 6.4, 10.2$ which are the points of maximum, zero and minimum amplitude of the magnetic field.

0, 300, 320, 380. Frames in the first row show the edf at $t = 0$ at the position where the magnetic field is maximum, zero and minimum. Next rows show the same at different times. At $t = 0$ we can see that the velocity in the y direction of the particles is greater than in the x direction, corresponding to our initial condition where, in the y direction the particles are hotter in the x direction. A deformation of the edf is observed from $t > 300$. We see that there is a x -dependent rotation of the edf at the positions of maximum and minimum magnetic field at each time. On the contrary there is net acceleration of electrons in the direction of the electrostatic force at the position of zero magnetic field. We also observe that with increase in time additional structures in velocity space are formed elongated along v_x . These additional structures in phase space may correspond to the particles that have been accelerated by the induced electric field and deflected by the magnetic field. To see further, we plot in Fig.3 in frames (A) and (B), the induced electric field vs time at $x = \pi/2k$ and $x = 3\pi/2k$, positions corresponding to first and second zero's in the magnetic field. We see that the induced electric field at first zero is positive while for the next it is negative, which relates to the acceleration of the particles as mentioned earlier. Same kind of distortions are observed in the corresponding contour plots in (C) and (D).

As shown earlier, for $t < 300$ the electrostatic field grows exponentially. In Fig.4, frames (A)-(B)-(C) show the time evolution of the electron distribution function versus v_x at fixed spatial

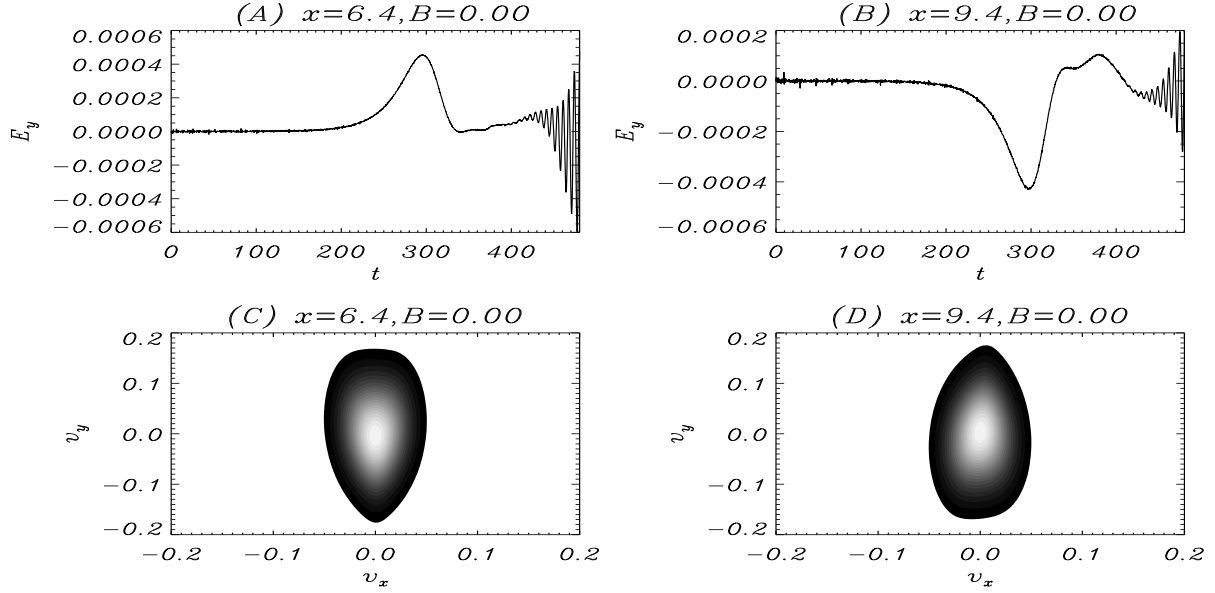


Figure 3. Frames (A, B) plot induced electric field E_y vs time at $x = 6.4, 9.4$ where $B_z = 0$. Frames (C, D) plot contour of edf in (v_x, v_y) phase plane at the same positions.

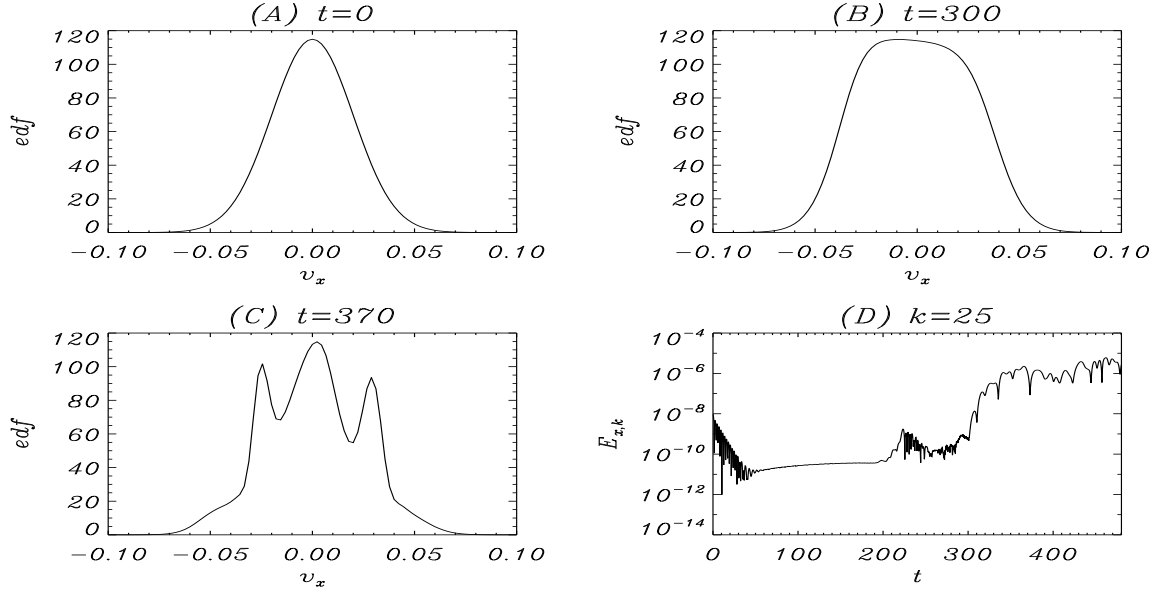


Figure 4. Frames (A,B,C) show the time evolution of edf versus v_x at $t = 0, 300, 370$. Frame (D) plots the time evolution of $E_{x,k}$ at $k = 25$.

position. We observe bumps along v_x direction of the distribution function at $t \geq 300$. These bumps remain for quite a long time up to $t = 370$ with a resonant velocity $v_r = 0.04$. In the last frame we show for corresponding $k=25$ the electrostatic field in time. We observe in the plot that indeed the electrostatic field grows in time during the period when we observe bumps in the edf.

4. Conclusions

With the help of the numerical simulations, we have shown the kinetic saturation of the Weibel instability. Furthermore, it has been shown that the evolution of the Weibel instability leads to distortions in the electron distribution function that give rise to the resonant onset of electrostatic field.

References

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